

CBCS SCHEME

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17MATDIP31

Third Semester B.E. Degree Examination, Jan./Feb. 2021 Additional Mathematics - I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1
- a. Find the modulus and amplitude of $\frac{(1+i)^2}{3+i}$. (06 Marks)
 - b. If $x + \frac{1}{x} = 2 \cos \alpha$, then prove that $x^n + \frac{1}{x^n} = 2 \cos n\alpha$. (07 Marks)
 - c. Find the fourth roots of $1 - i\sqrt{3}$ and represent them on an argand plane. (07 Marks)

OR

- 2
- a. If the vectors $2\hat{i} + \lambda\hat{j} + \hat{k}$ and $4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular to each other than find the value of λ . (06 Marks)
 - b. Find the sine of the angle between the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. (07 Marks)
 - c. Find λ such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar. (07 Marks)

Module-2

- 3
- a. Find the n^{th} derivative of $\cos x \cos 2x \cos 3x$. (06 Marks)
 - b. With usual notations prove that $\tan \phi = r \frac{d\theta}{dr}$. (07 Marks)
 - c. Prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{24} + \dots$ By using Maclaurin's expansion. (07 Marks)

OR

- 4
- a. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. (06 Marks)
 - b. If $u = f \left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x} \right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. (07 Marks)
 - c. If $u = e^x \cos y$, $v = e^x \sin y$, find $J = \frac{\partial(u, v)}{\partial(x, y)}$. (07 Marks)

Module-3

- 5
- a. Evaluate $\int_0^{\pi} x \cos^6 x \, dx$. (06 Marks)
 - b. Evaluate $\int_0^1 \int_0^1 \frac{dx dy}{\sqrt{(1-x^2)(1-y^2)}}$. (07 Marks)
 - c. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 y z \, dx \, dy \, dz$. (07 Marks)

OR

- 6 a. Evaluate $\int \sin^6 x \, dx$. (06 Marks)
- b. Evaluate $\iint_R (x^2 + y^2) \, dx \, dy$, where R is the triangle bounded by the lines $y = 0$, $y = x$ and $x = 1$. (07 Marks)
- c. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \, dx \, dy \, dz$. (07 Marks)

Module-4

- 7 a. A particle moves along a curve whose position vector is given by $\vec{r} = \left(t - \frac{t^3}{3}\right)\hat{i} + t^2\hat{j} + \left(t + \frac{t^3}{3}\right)\hat{k}$. Find the velocity and acceleration at $t = 3$. (06 Marks)
- b. Find the unit normal vector to the surface $xy + x + zx = 3$ at $(1, 1, 1)$. (07 Marks)
- c. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)

OR

- 8 a. A particle moves so that its position vector is given by $\vec{r} = \cos wt \hat{i} + \sin wt \hat{j}$, where w is a constant. Show that the velocity \vec{V} is perpendicular to \vec{r} . (06 Marks)
- b. If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$, show that $\vec{F} \text{ curl } \vec{F} = 0$. (07 Marks)
- c. Show that $\vec{f} = (\sin y + z)\hat{i} + (x \cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational. Also find ϕ such that $\vec{f} = \nabla\phi$. (07 Marks)

Module-5

- 9 a. Solve $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$. (06 Marks)
- b. Solve $\frac{dy}{dx} + y \cot x = \sin x$. (07 Marks)
- c. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$. (07 Marks)

OR

- 10 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$. (06 Marks)
- b. Solve $(y \cos x + \sin y + y) \, dx + (\sin x + x \cos y + x) \, dy = 0$. (07 Marks)
- c. Solve $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$. (07 Marks)
